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SOME APPLICATIONS OF THE RETRACTION THEOREM IN EXTERIOR ALGEBRA

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This paper considers examples of how the multiplicative structure of an ideal in an exterior algebra may be used to construct tensor invariants of the ideal. In particular, we consider the ideals arising from the study of decomposable elements of degree p, the relative theory of skew symmetric bilinear forms, and pencils of skew symmetric bilinear forms.

Let V be an n-dimensional vector space over a field $\underline{F}$, and V^* the dual vector space of linear functions on V. If W^* is a subspace of V^*, then the inclusion induces an inclusion on the exterior algebras

$$0 \to E(W^*) \to E(V^*) ,$$

and hence if J is an ideal in $E(W^*)$, this inclusion canonically induces an ideal I in $E(V^*)$. Matters being so we say that I is an extension of J, and J is a retraction of I.

Let $\langle \; , \; \rangle$ denote the canonical bilinear pairing between $E(V)$ and $E(V^*)$. This pairing allows us to introduce $\lrcorner$ as the adjoint of left exterior multiplication in $E(V)$ and $\llcorner$ as the adjoint of left exterior multiplication in $E(V^*)$ [1]. Then given an ideal I in $E(V^*)$, we may define

$$\operatorname{Char} I = \{X \in V \mid X \lrcorner I \subset I\} ,$$

the subspace of characteristic vectors of I, and its annihilator

$$C(I) = [\operatorname{Char} I]^{\perp} ,$$

the Cartan subspace of I.

The retraction theorem of Elie Cartan may now be stated as follows:

Theorem 1. *Let I be an ideal in $E(V^*)$, then $C(I)$ is the smallest subspace of V^* whose exterior algebra contains a retraction of I* [2].

The program is to utilize the existence of the Cartan subspace to construct tensor invariants from the multiplicative properties of a given ideal.

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