A FIXED POINT THEOREM OF DISCRETE GROUP ACTIONS ON RIEMANNIAN MANIFOLDS

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Abstract

We prove a fixed point theorem for a class of discrete group acting on manifolds of nonpositive curvature by isometry. These discrete groups include cocompact lattices in simply connected semisimple p-adic groups of rank at least two and large p. Hence it gives a geometric generalization of Margulis' superrigidity theorem for the Archimedean representation of these groups.

1. Introduction

Let N be a complete simply connected Riemannian manifold of nonpositive sectional curvature. N has a natural compactification $\bar{N} = N \cup \partial N$ by the sphere at infinity which is defined as the equivalent classes of geodesic rays [1]. Any group action on N by isometry extends to an action on $\bar{N}$.

Definition 1.1. A group Γ is said to have property (F) if any isometric action of Γ on any complete simply connected manifold of nonpositive sectional curvature N has a fixed point in $\bar{N}$.

In term of representations of Γ, property (F) has the following interpretation. Let H be a simple noncompact Lie group with trivial center. If Γ has property (F), then any homomorphism $\rho : \Gamma \mapsto H$ with Zariski dense image is precompact in H. This is because the symmetric space associated with H has nonpositive curvature and the image being Zariski