

**GEOMETRIC CRITERION FOR  
GIESEKER-MUMFORD STABILITY  
OF POLARIZED MANIFOLDS**

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In Geometric Invariant Theory, the notion of stability for any polarized projective variety is introduced. However to check the stability is usually a difficult problem; see [13], [7] and [21]. It is therefore very interesting to describe the meaning of stability by geometric data of the polarized projective varieties. In this paper we will in particular show that the Gieseker-Mumford stability of a polarized smooth projective variety (as used by them in [7], [13]) is related to the existence of a special metric on the polarized line bundle.

In early 80's, Yau conjectured the relation between notions of stability of manifolds and existence of special metrics such as Kähler-Einstein metrics. In this paper, we are working towards this direction and going to deal with the case of Gieseker-Mumford stability. Similar problems have been studied before. In Tian's recent work ([18], [19]), he dealt with the relation between Kähler-Einstein metric and stability. The notion of stability used by Tian is different from those used by Gieseker and Mumford. However we will see with modifications his methods can still be used in the study of the stability of polarized manifolds in the sense of Gieseker and Mumford.

Another motivation comes from the work on Mumford stability of vector bundles by Donaldson ([4],[5]), and by Uhlenbeck and Yau ([20]). They proved that Mumford stability is equivalent to the existence of

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