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# HITCHIN'S AND WZW CONNECTIONS ARE THE SAME

YVES LASZLO

## 1. Introduction

Let $X$ be an algebraic curve over the field $\mathbf{C}$ of complex numbers, which is assumed to be smooth, connected and projective. For simplicity, we assume that the genus of $X$ is $> 2$. Let $G$ be a simple simply connected group and $M_G(X)$ the coarse moduli scheme of semistable $G$-bundles on $X$. Any linear representation determines a line bundle $\Theta$ on $M$ and some nonnegative integer $l$ (the Dynkin index of the representation, cf [12], [13]). It is known that the choice of a (closed) point $x \in X(\mathbf{C})$ (and, a priori, of a formal coordinate near $x$) of $X$ determines an isomorphism (see 5) between the projective space of conformal blocks $\mathbf{P}B_l(X)$ (for $G$) of level $l$ and the space $\mathbf{P}H^0(M_G(X), \Theta)$ of generalized theta functions (see [3], [7],[12], [13]). In fact, it is observed in [20] that there is a coordinate free description of $B_l(X)$.

When the pointed curve $(X, x)$ runs over the moduli stack $\mathcal{M}_{g,1}$ of genus $g$ pointed curve, these 2 projective spaces organize in 2 projective bundles $\mathbf{P}\Theta$ and $\mathbf{PB}_l$. We first explain (see 5.7) how to identify these 2 projective bundles (this is a global version of the identification above). The projective bundle $\mathbf{P}\Theta$ has a canonical flat connection: the Hitchin connection [9] and $\mathbf{PB}_l$ has a flat connection, which we call the WZW connection coming from the conformal field theory (see [21] or [18]). In the rest of the paper, we prove that this canonical identification 5.7

$$\kappa : \ \mathbf{P}\Theta \stackrel{\sim}{\to} \mathbf{PB}_l$$

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