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NON-ZERO DEGREE MAPS TO HYPERBOLIC 3-MANIFOLDS

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Let us consider two closed, connected, orientable manifolds M, N with the same dimension n. Then, N is said to be *dominated* by M if there exists a non-zero degree map $f : M \longrightarrow N$. In this paper, we study the case where the dominated manifold N is hyperbolic. By an argument invoking the Gromov invariant, it is shown that the volume of N is bounded by a constant depending only on M; see Thurston [9, Chapter 6]. According to H.C. Wang [11], there are only finitely many hyperbolic n-manifolds with bounded volume if $n > 3$. This shows that the number of mutually non-homeomorphic, hyperbolic n-manifolds dominated by a fixed M is finite if $n \neq 3$. In the case of $n = 3$, a similar argument does not work. In fact, by Thurston's Hyperbolic Dehn Surgery Theorem [9], one can have infinitely many hyperbolic 3-manifolds with bounded volume, and hence Wang's theorem of dimension three does not hold. However, even in this case, we have the following theorem.

Theorem. *For any closed, connected, orientable* 3*-manifold* M*, the number of mutually non-homeomorphic, orientable, hyperbolic* 3*-manifolds dominated by* M *is finite.*

In [6], Reid and Wang proved the same assertion when M is hyperbolic and non-Haken by a method different from ours. We note that, by using some arguments in Boileau-Wang [2, §3], one can prove that this theorem does not hold when M is non-orientable. However, it would be impossible to apply their arguments to the orientable case; for example see Remark in §4. Moreover, our theorem is closely connected with Problem 3.100 by Y. Rong in [5], where he asked whether there are only

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