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MEASURED FOLIATIONS AND HARMONIC MAPS OF SURFACES

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1. Introduction

Fix a Riemannian surface of negative curvature (N^2, h) and a differentiable surface M_g^2 of the same genus g that will host various structures. Also fix a diffeomorphism $f_0 : M \to N$. It is well known ([4], [1], [21], [18]) that to every complex structure σ on M, there is a unique harmonic diffeomorphism $f(\sigma) : M(\sigma) \to (N, h)$ homotopic to $f_0 : M \to N$; one is led to consider what other, possibly ostensibly weaker, structures on M might also determine harmonic maps from M to N homotopic to f_0.

The goal of this paper is to show (Theorem 3.1) that a harmonic map $f(\sigma') : (M, \sigma') \to (N, h)$ may be uniquely specified by the initial choice of a class of measured foliations (representing the maximal stretch measured foliation for the harmonic map $f(\sigma')$) rather than an initial choice of complex structure: we observe that a measured foliation may be considered to be a differential-topological object in contrast to the analytical object that a complex structure σ represents.

Our proof has aspects of independent interest. In particular, in the proof of uniqueness (§4), from a harmonic map $f : (M^2, \sigma) \to (N, h)$ of a surface, we construct a naturally associated equivariant (area) minimal map $F : (\widetilde{M}^2, \tilde{\sigma}) \to (\widetilde{N}, \tilde{h}) \times (T, 2d)$ of the universal cover into the product of the universal cover $(\widetilde{N}, \tilde{h})$ with a real tree $(T, 2d)$. We show (Theorem 4.3) that for two dimensional negatively curved targets

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