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THE INVERSE SPECTRAL PROBLEM FOR SURFACES OF REVOLUTION

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Abstract

We prove that isospectral simple analytic surfaces of revolution are isometric.

0. Introduction

This article is concerned with the inverse spectral problem for metrics of revolution on S^2. We will assume that our metrics are real analytic and belong to a class $\mathcal{R}^*$ of rotationally invariant metrics which are of 'simple type' and satisfy some generic non-degeneracy conditions (see Definition (0.1)). In particular, we will assume they satisfy the generalized 'simple length spectrum' condition that the length functional on the loop space is a clean Bott-Morse function which takes on distinct values on distinct components of its critical set (up to orientation). Denoting by $Spec(S^2, g)$ the spectrum of the Laplacian Δ_g, our main result is the following:

Theorem I. *Spec:* $\mathcal{R}^* \to \mathbb{R}^{+\mathbb{N}}$ *is 1-1.*

Thus, if $(S^2, g), (S^2, h)$ are isospectral surfaces of revolution in $\mathcal{R}^*$, then g is isometric to h. It would be very desirable to strengthen this result by removing the assumption that $h \in \mathcal{R}^*$, thereby showing that metrics in $\mathcal{R}^*$ are spectrally determined within the entire class of analytic metrics on S^2 with simple length spectra. The only metric on S^2

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