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P-ADIC UNIFORMIZATION OF UNITARY SHIMURA VARIETIES. II

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Abstract

In this paper we show that certain Shimura varieties, uniformized by the product of complex unit balls, can be p-adically uniformized by the product of Drinfeld upper half-spaces and their equivariant coverings. We also extend a p-adic uniformization to automorphic vector bundles. It is a continuation of our previous work [38] and contains all cases (up to a central modification) of a uniformization by known p-adic symmetric spaces. The idea of the proof is to show that an arithmetic quotient of the product of Drinfeld upper half-spaces cannot be anything else than a certain unitary Shimura variety. Moreover, we show that difficult theorems of Yau and Kottwitz appearing in [38] may be avoided.

1. Introduction

Let M be a Hermitian symmetric domain (=Hermitian symmetric space of non-compact type), and let Δ be a torsion-free cocompact lattice in $\mathrm{Aut}(M)$. Then the quotient $\Delta\backslash M$ is a complex manifold, which has a unique structure of a complex projective variety Y_Δ (see [34, Ch. IX, §3]). A well-known theorem says that when Δ is an arithmetic congruence subgroup, the Shimura variety Y_Δ has a canonical structure over some number field E (see for example [22, II, Thm. 5.5]).

Let v be a prime of E. We are interested in a question whether Y_Δ can be p-adically (or more precisely v-adically) uniformized. By this we mean that the E_v-analytic space $(Y_\Delta \otimes_E E_v)^{an}$ is isomorphic to $\Delta\backslash\Omega$ for some E_v-analytic symmetric space Ω and some arithmetic group Δ, acting on Ω discretely. In the cases where a p-adic uniformization exists we are interested in the relation between M and Ω, Δ and Γ.

The main obstacle for attacking such a general problem is that there is no general definition of a p-adic symmetric space. The only p-adic

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