J. DIFFERENTIAL GEOMETRY
48 (1998) 531-555

TITS GEOMETRY ASSOCIATED WITH 4-DIMENSIONAL CLOSED REAL-ANALYTIC MANIFOLDS OF NONPOSITIVE CURVATURE

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Abstract

We investigate the geometry and topology of the Tits boundary associated with 4-dimensional closed, real-analytic manifolds of nonpositive curvature. We show that each homotopically nontrivial component is a union of geometric boundaries of flats in the corresponding Hadamard manifold and this can be used to describe the structure of its maximal dimensional quasi-flats. The homotopically trivial components are intervals of length smaller than π and we give a necessary and sufficient criterion for the existence of such intervals of length greater than zero.

Introduction

The Tits boundary $\partial_T X = (X(\infty), \mathrm{Td})$ of a complete, simply connected, nonpositively curved Riemannian manifold X is a metric space which reflects parts of the asymptotic geometry of X. In this paper we obtain a description of this space in the case that X is the universal covering of a compact real-analytic Riemannian manifold of nonpositive sectional curvature and dimension ≤ 4. While the situation in the 2- and 3-dimensional case is quite obvious and easy to describe (compare Section 1), new and interesting phenomena occur in dimension 4. Roughly speaking, up to dimension 3 the nontrivial components of $\partial_T X$

Received December 26, 1996, and, in revised form, October 16, 1997. The first author was supported by the Swiss National Science Foundation.

Keywords and phrases. nonpositive curvature, Tits geometry, higher rank subspaces

1991 *Mathematics Subject Classification.* 53C20, 53C23, 57M05.