J. DIFFERENTIAL GEOMETRY
48 (1998) 445-495

LIE GROUP VALUED MOMENT MAPS

ANTON ALEKSEEV, ANTON MALKIN
& ECKHARD MEINRENKEN

Abstract

We develop a theory of "quasi"-Hamiltonian G-spaces for which the moment map takes values in the group G itself rather than in the dual of the Lie algebra. The theory includes counterparts of Hamiltonian reductions, the Guillemin-Sternberg symplectic cross-section theorem and of convexity properties of the moment map. As an application we obtain moduli spaces of flat connections on an oriented compact 2-manifold with boundary as quasi-Hamiltonian quotients of the space $G^2 \times \cdots \times G^2$.

1. Introduction

The purpose of this paper is to study Hamiltonian group actions for which the moment map takes values not in the dual of the Lie algebra but in the group itself.

For the circle group S^1 this situation has been studied in literature (see e.g. [18], [24]). The standard example is the real 2-torus $T^2 = S^1 \times S^1$, with its standard area form and the circle acting by rotation of the first S^1; the moment map is given by projection to the second S^1. It is in fact known that every symplectic S^1-action on a symplectic manifold for which the 2-form has integral cohomology class admits an S^1-valued moment map.

In this paper we consider group valued moment maps for general non-abelian compact Lie groups G. One theory encountering group valued moment maps is the theory of Poisson-Lie group actions on symplectic manifolds [16], [17], where G is a Poisson Lie group and the target of the moment map is the dual Poisson-Lie group G^*. In [1] it was shown that for compact, connected, simply connected Lie groups G this theory is equivalent to the standard theory of Hamiltonian actions.

Received July 31, 1997.