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NODES ON SEXTIC HYPERSURFACES IN $\mathbb{P}^3$

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In this note we present a coding theory result which, together with Theorem 3.6.1 of [3], gives a short proof of a theorem of D. Jaffe and D. Ruberman:

Theorem [5]. *A sextic hypersurface in* $\mathbb{P}^3$ *has at most 65 nodes.*

W. Barth [1] has constructed an example with 65 nodes. Following V. Nikulin [7] and A. Beauville [2], one must limit the size of an even set of nodes, and then prove a result about binary linear **codes** (i.e., linear subspaces of $\mathbb{F}^n$, where $\mathbb{F}$ is the field of two elements). The first step is the aforementioned result of Casnati–Catanese:

Theorem [3]. *On a sextic hypersurface, an even set of nodes has cardinality 24, 32 or 40.*

The desired theorem will follow from:

Theorem A. *Let* $V \subset \mathbb{F}^{66}$ *be a code, with weights from among 24, 32 and 40. Then* $\dim(V) \leq 12$.

1. Codes from nodal hypersurfaces

(1.1) Let $\Sigma \subset \mathbb{P}^3$ be a hypersurface of degree d having only μ ordinary double points as singularities. Let $\pi : S \to \Sigma$ be the minimal resolution of the singularities, with exceptional (-2)-curves E_i. Thus

$$E_i \cdot E_j = -2\delta_{ij}. \tag{1.1.1}$$

S is diffeomorphic to a smooth hypersurface of degree d.

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