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SUTURED MANIFOLD HIERARCHIES, ESSENTIAL LAMINATIONS, AND DEHN SURGERY

YING-QING WU

0. Introduction

A compact orientable surface F with nonnegative Euler characteristic is either a sphere, a disk, a torus, or an annulus. If a 3-manifold M contains such an essential surface, then it is said to be reducible, ∂-reducible, toroidal, or annular, respectively. Any such surface can be used to decompose the manifold further into simpler manifolds. We say that M is a *simple manifold* if it has no such surfaces. A simple manifold is expected to have a nice geometric structure. If M has nonempty boundary, then the Geometrization Theorem of Thurston for Haken manifolds says that M with boundary tori removed admits a finite volume hyperbolic structure with totally geodesic boundary. When M has no boundaries, Thurston's Geometrization Conjecture asserts that M is either hyperbolic, or is a Seifert fiber space with orbifold a sphere with at most 3 cone points.

Suppose T is a torus boundary component of M. We use $M(\gamma)$ to denote the manifold obtained by Dehn filling on T so that the slope γ on T bounds a disk in the Dehn filling solid torus. When $M = E(K)$ is the exterior of a knot K in S^3, denote $M(\gamma)$ by $K(\gamma)$, and call it the manifold obtained by γ surgery on the knot K. It is well known that if M is simple, then there are only finitely many Dehn fillings on

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