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RIGIDITY OF HYPERBOLIC CONE-MANIFOLDS AND HYPERBOLIC DEHN SURGERY

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The local rigidity theorem of Weil [28] and Garland [12] for complete, finite volume hyperbolic manifolds states that there is no non-trivial deformation of such a structure through *complete* hyperbolic structures if the manifold has dimension at least 3. If the manifold is closed, the condition that the structures be complete is automatically satisfied. However, if the manifold is non-compact, there may be deformations through incomplete structures. This cannot happen in dimensions greater than 3 (Garland-Raghunathan [13]); but there are always non-trivial deformations in dimension 3 (Thurston [24]) in the non-compact case.

In this paper, we extend this rigidity and deformation theory to a class of finite volume, orientable 3-dimensional hyperbolic cone-manifolds, *i.e.*, hyperbolic structures on 3-manifolds with cone-like singularities along a knot or link. Our main result is that such structures are locally rigid if the cone angles are fixed, under the extra hypothesis that all cone angles are at most 2π. We can view the singular structure as an incomplete, smooth structure on the complement of the singular locus whose metric completion is the singular cone structure. The space of deformations of structures on this open manifold has non-zero dimension, so there will be deformations without the condition that the cone angles remain fixed. We show that it is possible to deform the structure so that the metric completion is still a cone-manifold, and that one

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