CASSON INVARIANT, SIGNATURE DEFECT OF FRAMED MANIFOLDS AND THE SECONDARY CHARACTERISTIC CLASSES OF SURFACE BUNDLES

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1. Introduction

The purpose of the present paper is twofold. First we relate the Casson invariant to the signature defect of Hirzebruch, which is a fine invariant defined for framed manifolds. Secondly we introduce the notion of the *secondary* characteristic classes of surface bundles. This consists of a series of cohomology classes of a certain subgroup of the mapping class group of orientable surfaces. The definition is given by first examining the above relation between the Casson invariant and the signature defect precisely and then generalizing it to higher dimensional context.

The Casson invariant is an integer valued invariant defined for oriented homology 3-spheres. Roughly speaking it counts the number (with signs) of conjugacy classes of irreducible representations of the fundamental group of an oriented homology 3-sphere into the Lie group $SU(2)$. In the cases where the homology 3-spheres are given as the results of performing $1/n$ Dehn surgeries along knots in the 3-dimensional sphere S^3, there is a formula which expresses their Casson invariants in terms of a classical invariant of knots, namely the Alexander polynomial.

In our previous papers [22], [23], we investigated how the Casson invariant is related to the structure of the mapping class group of oriented surfaces through the correspondence between elements of the mapping class group and 3-manifolds via the Heegaard splittings. Here we briefly