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CONTINUOUS FAMILIES OF ISOSPECTRAL RIEMANNIAN METRICS WHICH ARE NOT LOCALLY ISOMETRIC

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Introduction

Two compact Riemannian manifolds are said to be *isospectral* if the associated Laplace-Beltrami operators, acting on smooth functions, have the same eigenvalue spectrum. If the manifolds have boundary, we specify *Dirichlet* or *Neumann* isospectrality depending on the boundary conditions imposed on the eigenfunctions.

Numerous examples of isospectral compact manifolds have been constructed; see, for example, [4], [5], [7], [12], [13], [14], [15], [16], [17], [19], [24] and [26] or the survey articles [1], [2], [6], and [9]. Until recently however, all known examples of isospectral manifolds were locally isometric, though not globally isometric. In particular, the closed isospectral manifolds had a common cover. Then Z. Szabó [25] gave a construction of pairs of isospectral compact manifolds with boundary which are not locally isometric, and the first author [10], [11] constructed pairs of isospectral closed Riemannian manifolds which are not locally isometric. Szabo pointed out that the curvature operators of these isospectral manifolds have different eigenvalues, thus identifying a specific local invariant which is not spectrally determined.

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