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HARMONIC MAPS OF INFINITE ENERGY AND RIGIDITY RESULTS FOR REPRESENTATIONS OF FUNDAMENTAL GROUPS OF QUASIPROJECTIVE VARIETIES

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Abstract

We show the existence of harmonic maps associated with reductive homomorphisms of the fundamental group of a quasiprojective variety into a linear algebraic group over an archimedean or p-adic field. The map we construct may have infinite energy, but it satisfies suitable estimates at infinity, and it is pluriharmonic. We use this map to complete a previous result of Jost-Yau [21] on strong rigidity of nonuniform lattices in Hermitian symmetric spaces, and to drop a topological restriction in our previous theory ([24], [25]) of representations of fundamental groups of quasiprojective varieties.
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Introduction

Hodge theory represents cohomology classes by harmonic forms, and it is a fundamental tool in Kähler geometry. Homology or cohomology groups, however, do not contain the complete topological information about a manifold X, and, in particular, the fundamental group may be much more complicated than the first homology group, its abelianized version, indicates. Via the Albanese period mapping, the fundamental group acts as a lattice on some vector space. This can be considered as an abelian representation of $\pi_1(X)$, yielding the first homology group. It is therefore natural to study also nonabelian representations of $\pi_1(X)$, in order to obtain further information about the topology of X. In the same way as the Albanese period map is harmonic (which is essentially

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