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EXACT LAGRANGE SUBMANIFOLDS, PERIODIC ORBITS AND THE COHOMOLOGY OF FREE LOOP SPACES

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Abstract

We prove that there are obstructions to the existence of an exact Lagrange embedding from a closed manifold L to T^*N. This may be seen as an extension of Gromov's theorem as formulated by Lalonde and Sikorav, showing that no such embedding exists for N open. For example we answer positively a question by Lalonde and Sikorav on the non-existence of exact Lagrange embeddings from T^2 into T^*S^2. Our obstruction is in terms of the cohomology of the loop space of L and N and the map induced by the embedding in the cohomologies of these loop spaces. In particular, we give obstructions to the existence of an exact Lagrangian embedding inducing a degree-zero map from L to N. As another application of our method, we prove the Weinstein conjecture in cotangent bundles of simply connected manifolds (removing an assumption in a previous joint paper with H. Hofer). A number of these results had been announced in [48] and [49]

0. Introduction

Let N be a manifold, T^*N its cotangent bundle, endowed with the standard symplectic form, $\omega = d\lambda$ where $\lambda = \sum_{i=1}^n p_i dq^i$ in local coordinates (the q^i are coordinates on N, and the p_i the dual coordinates).

An embedding from a manifold L of dimension $n = dim(L) = dim(N)$ to T^*N is said to be Lagrange if ω vanishes on the tangent space to L, and exact (Lagrange) if λ induces an exact form on L.

It is one of the striking results of [19], that there are no exact Lagrange embeddings from a compact manifold L into $M = V \times \mathbb{R}$, and in fact, as noticed by Lalonde and Sikorav ([28]), Gromov's argument

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