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GRAFTING, HARMONIC MAPS AND PROJECTIVE STRUCTURES ON SURFACES

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Abstract

Grafting is a surgery on Riemann surfaces introduced by Thurston; it connects hyperbolic geometry and the theory of projective structures on surfaces. ([4], [7]) We will discuss the space of projective structures in terms of the Thurston's geometric parametrization given by grafting. From this approach we will prove that on any compact Riemann surface with genus greater than 1 there exist infinitely many projective structures with Fuchsian holonomy representations. In course of the proof it will turn out that grafting is closely related to harmonic maps between surfaces.

1. Introduction

A projective structure (or a $\mathbf{C}P^1$-structure) on a surface is a coordinate system modelled on the projective space $\mathbf{C}P^1$ such that the transition maps are projective homeomorphisms (and hence the restriction of elements of $\mathrm{PSL}(2,\mathbf{C})$). For an oriented closed surface Σ_g of genus $g \geq 2$, it is well known that the space of projective structures P_g on Σ_g is parametrized by the bundle of holomorphic quadratic differentials on Riemann surfaces $\pi : Q_g \to T_g$ over the Teichmüller space: for each projective structure on Σ_g, taking the Schwarzian derivative of the developing map we have a quadratic differential which is holomorphic with respect to the underlying complex structure of the projective structure. As this parametrization is dealing with projective or complex analytic mappings and manifolds, a lot of researches have been developed from

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