

## A CHARACTERIZATION OF RIEMANN'S MINIMAL SURFACES

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### Abstract

We prove that Riemann's minimal surfaces are the only properly embedded minimal tori with two planar ends in  $\mathbb{R}^3/T$ , where  $T$  is the group generated by a nontrivial translation in  $\mathbb{R}^3$ . In the proof of this result we find all the properly immersed minimal tori with two parallel embedded planar ends. The space of such surfaces is described by regular curves, parameterized by  $\mathbb{R}$ , in the moduli space of conformal structures on a topological torus. Except in the case of Riemann's minimal surfaces, these curves contain points which yield minimal surfaces with vertical flux, and hence the surfaces are not embedded.

### Introduction

In a paper published in 1867, Riemann [21] found a one-parameter family of complete, embedded, singly-periodic minimal surfaces foliated by circles and lines in parallel planes. These surfaces, known nowadays as Riemann examples or Riemann's minimal surfaces, were characterized by Riemann in [21] as the only minimal surfaces fibered by circles in parallel planes besides the catenoid. Since then many different characterization results have been proved for the surfaces.

Enneper [6] proved around 1870 that a minimal surface fibered by circular arcs was in fact a piece of a Riemann example or a piece of the catenoid. Shiffman [23] proved in 1956 that a minimal annulus spanning

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