THE GEOMETRY OF THE SPACE OF
HOLOMORPHIC MAPS FROM A RIEMANN
SURFACE TO A COMPLEX PROJECTIVE SPACE

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0. Introduction

In recent years there have been a number of papers on the homology and geometry of spaces of holomorphic maps of the Riemann sphere into complex varieties, [21], [5], [17], [18], [3], [9]. However, the very classic question of the structure of the spaces of holomorphic maps from complex curves $M_g$ of genus $g \geq 1$ to complex varieties has proved to be very difficult. There is Segal's stability theorem, [21], which shows that the natural inclusion of the space of based holomorphic maps of degree $k$ into the space of all based maps $\text{Hol}_k^*(M_g, V) \hookrightarrow \text{Map}_k^*(M_g, V)$ is a homotopy equivalence through a range of dimensions which increases with $k$ when $V = \mathbb{P}^n$, the complex projective space. Also, there is the extension of this result by J. Hurtubise to further $V$; [11]. But that is about all.

In this paper we begin the detailed study of the topology of the $\text{Hol}_k^*(M_g, \mathbb{P}^n)$. We are able to completely determine these spaces and their homology when $M_g$ is an elliptic curve, and we give an essentially complete determination in the case where $M_g$ is hyperelliptic. In particular we determine the rational homology of these spaces when $k \geq 2g-1$ in the elliptic and hyperelliptic cases.

Let $M_g$ be a genus $g$ complex curve. The key analytic result on the structure of $\text{Hol}_k^*(M_g, \mathbb{P}^1)$ is Abel's Theorem which identifies the