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THE GEOMETRY OF THE SPACE OF HOLOMORPHIC MAPS FROM A RIEMANN SURFACE TO A COMPLEX PROJECTIVE SPACE

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0. Introduction

In recent years there have been a number of papers on the homology and geometry of spaces of holomorphic maps of the Riemann sphere into complex varieties, [21], [5], [17], [18], [3], [9]. However, the very classic question of the structure of the spaces of holomorphic maps from complex curves M_g of genus $g \geq 1$ to complex varieties has proved to be very difficult. There is Segal's stability theorem, [21], which shows that the natural inclusion of the space of based holomorphic maps of degree k into the space of all based maps $Hol_k^*(M_g, V) \hookrightarrow Map_k^*(M_g, V)$ is a homotopy equivalence through a range of dimensions which increases with k when $V = \mathbf{P}^n$, the complex projective space. Also, there is the extension of this result by J. Hurtubise to further V; [11]. But that is about all.

In this paper we begin the detailed study of the topology of the $Hol_k^*(M_g, \mathbf{P}^n)$. We are able to completely determine these spaces and their homology when M_g is an elliptic curve, and we give an essentially complete determination in the case where M_g is hyperelliptic. In particular we determine the rational homology of these spaces when $k \geq 2g-1$ in the elliptic and hyperelliptic cases.

Let M_g be a genus g complex curve. The key analytic result on the structure of $Hol_k^*(M_g, \mathbf{P}^1)$ is Abel's Theorem which identifies the

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