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LAMINATIONS IN HOLOMORPHIC DYNAMICS

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1. A missing line in the dictionary

There is an intriguing dictionary between two branches of conformal dynamics: the theory of Kleinian groups and dynamics of rational maps. This dictionary was introduced by Sullivan, and led him in the early 80's to the no wandering domains theorem, deformation theory and geometric measure theory for holomorphic maps. Thurston's rigidity and realization theory, developed at the same time, was also motivated by this analogy. More recently, McMullen has made important contributions to the renormalization theory motivated by the analogy with 3-manifolds which fiber over the circle [32], [33].

However, the translation from one language to another, as in usual life, is not automatic. There are concepts and methods in each of these fields which only barely allow translation to the other one. And even when it is possible, the results achieved are often complementary (see Sullivan's table in [46] of the results on the structural stability and hyperbolicity problems).

In this paper we explore a construction which attempts to provide an element of the dictionary that has so far been missing: an explicit object that plays for a rational map the role played by the hyperbolic 3-orbifold quotient of a Kleinian group. To build this object we replace the notion of manifold by "lamination", which is a topological object whose local structure is the product of Euclidean space by a (possibly complicated) transverse space.

Another goal of this work is to study the space of backward orbits of a rational function. Since Fatou and Julia, inverse branches of iterated

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