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AN ESTIMATE FOR THE GAUSS CURVATURE OF MINIMAL SURFACES IN $\mathbf{R}^m$ WHOSE GAUSS MAP OMITS A SET OF HYPERPLANES

ROBERT OSSERMAN & MIN RU

1. Introduction

The purpose of this paper is to prove the following theorem.

Theorem 1.1 (Main Theorem). *Let $x : M \to \mathbf{R}^m$ be a minimal surface immersed in $\mathbf{R}^m$. Suppose that its generalized Gauss map g omits more than $\frac{m(m+1)}{2}$ hyperplanes in $\mathbf{P}^{m-1}(\mathbf{C})$, located in general position. Then there exists a constant C, depending on the set of omitted hyperplanes, but not the surface, such that*

$$|K(p)|^{1/2} d(p) \leq C, \tag{1}$$

where $K(p)$ is the Gauss curvature of the surface at p, and $d(p)$ is the geodesic distance from p to the boundary of M.

This theorem provides a considerable sharpening of an earlier result of the same type:

Theorem 1.2 (Osserman [12]). *An inequality of the form (1) holds for all minimal surfaces in $\mathbf{R}^m$ whose Gauss map omits a neighborhood of some hyperplane in $\mathbf{P}^{m-1}(\mathbf{C})$.*

Also, Theorem 1.1 implies the earlier result:

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