

SYMPLECTIC TOPOLOGY AS THE
GEOMETRY OF ACTION FUNCTIONAL. I
—RELATIVE FLOER THEORY ON
THE COTANGENT BUNDLE

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1. Introduction and the main result

In the late 70's or the beginning of the 80's, Eliashberg proved the following theorem, which first indicated the existence of *symplectic topology* that is supposed to be finer than *differential topology*.

C^0 -rigidity theorem [Eliashberg]. *The group $\text{Symp}_\omega(P)$ of symplectic diffeomorphisms on a symplectic manifold (P, ω) is C^0 -closed in $\text{Diff}(P)$.*

Eliashberg's original proof [12] relies on a structure theorem on the *combinatorial* structure of the *wave front set* of certain Legendrian submanifolds in the one-jet bundle. The complete detail of the proof of this structure theorem, however, has not been published in the literature. The heart of his proof is some kind of non-squeezing theorem, which he proved using the above structure theorem. In a seminal paper [28] in 1985, Gromov introduced the *elliptic techniques of pseudo-holomorphic curves* and proved, among many other things, the following non-squeezing theorem.

Non-squeezing theorem [Gromov]. *Let $B^{2n}(R) \subset \mathbb{C}^n$ be the standard R -ball in \mathbb{C}^n and w_0 be the canonical symplectic structure on \mathbb{C}^n . Then there is a symplectic embedding*

$$\phi : (B^{2n}(R), w_0) \rightarrow (Z^{2n}(r), w_0)$$

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