INTERIOR ESTIMATES AND LONGTIME SOLUTIONS FOR MEAN CURVATURE FLOW OF NONCOMPACT SPACELIKE HYPERSURFACES IN MINKOWSKI SPACE

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Introduction

Spacelike hypersurfaces with prescribed mean curvature have played a major role in the study of Lorentzian manifolds. Maximal (mean curvature zero) hypersurfaces were used in the first proof of the positive mass theorem ([17]). Constant mean curvature hypersurfaces provide convenient time gauges for the Einstein equations ([7]). For a survey of results we refer to [3].

In [5] and [6], it was shown that entire solutions of the maximal surface equation

$$H(u) = \operatorname{div}\left(\frac{Du}{\sqrt{1 - |Du|^2}} \right) = 0$$

for spacelike hypersurfaces in Minkowski space are linear. The proof of this remarkable result is based on an interior a priori estimate for the gradient function

$$v = \frac{1}{\sqrt{1 - |Du|^2}}.$$

In fact, estimates for this quantity form the basis of existence proofs for spacelike hypersurfaces with prescribed mean curvature functions in a variety of contexts. These surfaces are described by nonlinear elliptic