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MINIMAL LAGRANGIAN DIFFEOMORPHISMS AND THE MONGE-AMPÈRE EQUATION

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0. Introduction

In this paper we consider two problems: one from geometry, one from analysis.

Consider, here and throughout this paper, two connected, simply connected, closed, bounded domains D_1 and D_2 in $\mathbb{R}^2$ with smooth boundaries. Suppose that the domains have equal area. It is well-known that there exists an area-preserving diffeomorphism $\psi : D_1 \to D_2$ which is smooth up to the boundary. (For a discussion of this and related questions see [**?**]). However, the differential equations which determine ψ form an underdetermined system and hence ψ cannot be expected to closely reflect the geometry of the domains D_1 and D_2. Consequently, it is an interesting problem to find further conditions on an area preserving diffeomorphism to more tightly link the diffeomorphism to the geometry of the domains.

Such a condition is suggested by the following theorem of R. Schoen [**?**] and, independently, F. Labourie [**?**]. Let M be a compact Riemann surface of genus $g \geq 2$. Let g_1, g_2 be a pair of hyperbolic metrics on M. We say a map $u : (M, g_1) \to (M, g_2)$ is a minimal map if the graph of u is a minimal surface in $M \times M$.

Theorem 0.1. *There is a unique, area preserving, minimal map* $u : (M, g_1) \to (M, g_2)$ *homotopic to the identity.*

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