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CONTACT CIRCLES ON 3-MANIFOLDS

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A *contact circle* on a 3-manifold is a pair of contact forms that defines a linear circle of contact forms (see below for the formal definition). This concept was introduced in [6], and there we gave a complete classification of those 3-manifolds that admit a contact circle satisfying a certain additional volume constraint.

In the present paper, whose methods are independent of those employed in [6], we show that every (closed, orientable) 3-manifold admits a contact circle.

1. Outline

We begin by recalling the precise definition of a contact circle.

Definition 1.1. A *contact circle* on a 3-manifold is a pair of contact forms (ω_1, ω_2) such that any non-trivial linear combination $\lambda_1\omega_1 + \lambda_2\omega_2$ with constant coefficients $(\lambda_1, \lambda_2) \neq (0,0)$ is again a contact form.

In other words, we call a pair of 1-forms (ω_1, ω_2) a contact circle if

$$(\lambda_1\omega_1 + \lambda_2\omega_2) \wedge (\lambda_1\, d\omega_1 + \lambda_2\, d\omega_2)$$

is a volume form for all $(\lambda_1, \lambda_2) \neq (0,0)$, and it clearly suffices to check this condition for (λ_1, λ_2) with $\lambda_1^2 + \lambda_2^2 = 1$, hence the name.

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