

RATIONAL BLOWDOWNS OF SMOOTH 4-MANIFOLDS

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1. Introduction

The invariants of Donaldson and of Seiberg and Witten are powerful tools for studying smooth 4-manifolds. A fundamental problem is to determine procedures which relate smooth 4-manifolds in such a fashion that their effect on both the Donaldson and Seiberg-Witten invariants can be computed. The purpose of this paper is to initiate this study by introducing a surgical procedure, called rational blowdown, and to determine how this procedure affects these two sets of invariants. The technique of rationally blowing down and its effect on the the Donaldson invariant were first announced at the 1993 Georgia International Topology Conference and represents the bulk of the mathematics in this paper. We fell upon this surgical procedure while we were investigating the behavior of the Donaldson invariant in the presence of embedded spheres and while investigating methods for producing a topological logarithmic transform. The rational blowdown procedure completes the full computation of the Donaldson series (and Seiberg-Witten invariants) of all elliptic surfaces with $p_g \geq 1$ by showing that the Donaldson series of elliptic surfaces is that conjectured by Kronheimer and Mrowka in [23]:

Theorem 1.1. *Let $E(n; p, q)$, $n \geq 2$, be the simply connected elliptic surface with $p_g = n - 1$ and multiple fibers of relatively prime orders*

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