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RIGIDITY IN THE HARMONIC MAP HEAT FLOW

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Abstract

We establish various uniformity properties of the harmonic map heat flow, including uniform convergence in L^2 exponentially as $t \to \infty$, and uniqueness of the positions of bubbles at infinite time. Our hypotheses are that the flow is between 2-spheres, and that the limit map and any bubbles share the same orientation.

1. Introduction

Let us consider smooth maps $\phi : S^2 \to S^2$. We use $z = x + iy$ as a complex coordinate on the domain, obtained by stereographic projection, and write the metric as $\sigma^2 dz d\bar{z}$, where

$$\sigma(z) = \frac{2}{1+|z|^2}.$$

Similarly we have a coordinate u on the target, and a metric $\rho^2 du d\bar{u}$. We are using the notation

$$dz = dx + idy, \qquad d\bar{z} = dx - idy,$$

with analogues for du and $d\bar{u}$, and we will write

$$u_z = \frac{1}{2}(u_x - iu_y), \qquad u_{\bar{z}} = \frac{1}{2}(u_x + iu_y).$$

To the map ϕ we associate the energy densities

$$e_\partial(\phi) = \frac{\rho^2(u)}{\sigma^2}|u_z|^2, \qquad e_{\bar{\partial}}(\phi) = \frac{\rho^2(u)}{\sigma^2}|u_{\bar{z}}|^2,$$

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