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BIRATIONAL SYMPLECTIC MANIFOLDS AND THEIR DEFORMATIONS

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1. Introduction

Compact complex manifolds X^{2n} with holonomy group $Sp(n)$ can algebraically be characterized as simply connected compact Kähler manifolds with a unique (up to scalars) holomorphic symplectic two-form ([2]). These manifolds, which are higher-dimensional analogues of K3 surfaces, are called irreducible symplectic.

Beauville was able to generalize the local Torelli theorem, one of the fundamental results in the theory of K3 surfaces, to all irreducible symplectic manifolds. His results show that there exists a (coarse) moduli space $\mathcal{M}$ of marked irreducible symplectic manifolds and that the period map

$$P : \mathcal{M} \to \mathbb{P}(\Gamma \otimes \mathbb{C})$$

is étale over $Q \subset \mathbb{P}(\Gamma \otimes \mathbb{C})$ – an open subset of a quadric defined by $q(x) = 0$ and $q(x + \bar{x}) > 0$. By definition, a marking is an isomorphism of lattices $\sigma : H^2(X, \mathbb{Z}) \cong \Gamma$, where $H^2(X, \mathbb{Z})$ is endowed with the quadratic form defined in [2] and Γ is a fixed lattice.

For K3 surfaces the moduli space $\mathcal{M}$ consists of two connected components which can be identified by $(X, \sigma) \mapsto (X, -\sigma)$. The global Torelli theorem for K3 surfaces asserts that the period map P restricted to either of the two components, say $\mathcal{M}_0$, is surjective and 'almost injective'. More precisely, if (X, σ) and (X', σ') are two points in $P_0^{-1}(x)$, then (X, σ), $(X', \sigma') \in \mathcal{M}_0$ are non-separated and the underlying X and X' are isomorphic K3 surfaces containing at least one (-2)-curve. Furthermore, for $x \in Q$ in the complement of the union of countably

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