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EXOTIC DEFORMATION QUANTIZATION

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1. Introduction

Let $\mathcal{A}$ be one of the following commutative associative algebras: the algebra of all smooth functions on the plane: $\mathcal{A} = C^\infty(\mathbf{R}^2)$, or the algebra of polynomials $\mathcal{A} = \mathbf{C}[p,q]$ over $\mathbf{R}$ or $\mathbf{C}$. There exists a non-trivial formal *associative* deformation of $\mathcal{A}$ called the *Moyal $\star$-product* (or the standard $\star$-product). It is defined as an associative operation $\mathcal{A}^{\otimes 2} \to \mathcal{A}[[\hbar]]$ where $\hbar$ is a formal variable. The explicit formula is:

$$F \star_\hbar G = FG + \sum_{k\geq 1} \frac{(i\hbar)^k}{2^k k!}\{F,G\}_k, \tag{1}$$

where $\{F,G\}_1 = \dfrac{\partial F}{\partial p}\dfrac{\partial G}{\partial q} - \dfrac{\partial F}{\partial q}\dfrac{\partial G}{\partial p}$ is the standard Poisson bracket, and the higher order terms are:

$$\{F,G\}_k = \sum_{i=0}^{k}(-1)^i \begin{pmatrix} k \\ i \end{pmatrix} \frac{\partial^k F}{\partial p^{k-i}\partial q^i}\frac{\partial^k G}{\partial p^i \partial q^{k-i}}. \tag{2}$$

The Moyal product is the *unique* (modulo equivalence) non-trivial formal deformation of the associative algebra $\mathcal{A}$ (see [13]).

Definition 1. A formal associative deformation of $\mathcal{A}$ given by formula (1) is called a *$\star$-product* if the following hold:

1) the first order term coincides with the Poisson bracket: $\{F,G\}_1 = \{F,G\}$;

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