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ON THE INTEGRABLE GEOMETRY OF SOLITON EQUATIONS AND N=2 SUPERSYMMETRIC GAUGE THEORIES

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Abstract

We provide a unified construction of the symplectic forms which arise in the solution of both N=2 supersymmetric Yang-Mills theories and soliton equations. Their phase spaces are Jacobian-type bundles over the leaves of a foliation in a universal configuration space. On one hand, imbedded into finite-gap solutions of soliton equations, these symplectic forms assume explicit expressions in terms of the auxiliary Lax pair, expressions which generalize the well-known Gardner-Faddeev-Zakharov bracket for KdV to a vast class of 2D integrable models; on the other hand, they determine completely the effective Lagrangian and BPS spectrum when the leaves are identified with the moduli space of vacua of an N=2 supersymmetric gauge theory. For SU(N_c) with $N_f \leq N_c+1$ flavors, the spectral curves we obtain this way agree with the ones derived by Hanany and Oz and others from physical considerations.

I. Introduction

A particularly striking aspect of the recent solutions of N=2 supersymmetric Yang-Mills theories [1] -[4] has been the emergence of integrable structures [5] - [6], structures which had surfaced in the completely different context of soliton equations and their Whitham-averaged counterparts [7], [9], [10]. On the gauge theory side, the moduli space of inequivalent vacua is identified with a moduli space of certain compact Riemann surfaces Γ, and both the effective Lagrangian and the Bogomolny-Prasad-Sommerfeld spectrum can be read off from the periods of a *meromorphic* 1-form $d\lambda$ on Γ. A defining property of $d\lambda$ is that its external derivative $\delta d\lambda$ be a *holomorphic* symplectic form ω

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