J. DIFFERENTIAL GEOMETRY
45 (1997) 336-348

NILPOTENT GROUPS AND UNIVERSAL COVERINGS OF SMOOTH PROJECTIVE VARIETIES

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1. Introduction

Characterizing the universal coverings of smooth projective varieties is an old and hard question. Central to the subject is a conjecture of Shafarevich according to which the universal cover $\widetilde{X}$ of a smooth projective variety is holomorphically convex, meaning that for every infinite sequence of points without limit points in $\widetilde{X}$ there exists a holomorphic function unbounded on this sequence.

In this paper we try to study the universal covering of a smooth projective variety X whose fundamental group $\pi_1(X)$ admits an infinite image homomorphism

$$\rho : \pi_1(X) \longrightarrow L$$

into a complex linear algebraic group L. We will say that a nonramified Galois covering $X' \to X$ corresponds to a representation $\rho : \pi_1(X) \to L$ if its group of deck transformations is $\mathrm{im}(\rho)$.

Definition 1.1. We call a representation $\rho : \pi_1(X) \to L$ linear, reductive, solvable or nilpotent if the Zariski closure of its image is a linear, reductive, solvable or nilpotent algebraic subgroup in L. We call the corresponding covering linear, reductive, solvable or nilpotent respectively.

The natural homomorphism $\pi_1(X,x) \to \hat{\pi}_{\mathrm{uni}}(X,x)$ to Malcev's pro-unipotent completion will be called the Malcev representation and the corresponding covering the Malcev covering.

Received December 1, 1995, and, in revised form, March 3, 1996. The author was partially supported by A.P. Sloan Dissertational Fellowship