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A SPLITTING THEOREM FOR ISOPARAMETRIC SUBMANIFOLDS IN HILBERT SPACE

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1. Introduction

Recall that for a submanifold M of a Hilbert space V, the *end point map* $\eta : \nu(M) \longrightarrow V$ is defined by $\eta(v) = x + v$ for $v \in \nu(M)_x$, where $\nu(M)$ is the normal bundle of M. M is called *proper Fredholm* if it has finite codimension and the end point map restricted to any finite normal disk bundle is a proper Fredholm map. A proper Fredholm submanifold M is called *isoparametric* if its normal bundle is globally flat and the shape operators along any parallel normal vector field are conjugate. We will always assume that M is complete. An isoparametric submanifold M of V is called *decomposable* if there exist two proper closed affine subspaces V_1, V_2 of V and isoparametric submanifolds M_i in V_i for $i = 1, 2$ such that $V = V_1 \oplus V_2$ and $M = M_1 \times M_2$, otherwise, M is called *indecomposable*. To every isoparametric submanifold, there is a cannonical way to associate a Coxeter group (cf. [10]). A Coxeter group is called *decomposable* if its Coxeter diagram is not connected. The main purpose of this paper is to prove the following decomposition theorem which was conjectured in [4].

Theorem A. *An isoparametric submanifold is decomposable if its Coxeter group is decomposable.*

It is easy to see that if an isoparametric submanifold M is decomposed into the product of two isoparametric submanifolds and each component has nontrivial Coxeter group, then the Coxeter group of M is decomposable. Note that an isoparametric submanifold has trivial Coxeter group if and only if it is a closed affine subspace of the ambient

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