J. DIFFERENTIAL GEOMETRY
45 (1997) 257-272

LIMITS OF SOLUTIONS TO THE KÄHLER-RICCI FLOW

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1. Introduction

We consider the Kähler-Ricci flow

$$\frac{\partial}{\partial t} g_{i\bar{j}} = -R_{i\bar{j}} \tag{1.1}$$

on a complex manifold X. Following [5], we have

Definiton 1.1. A complete solution $g_{i\bar{j}}$ to Eq.(1.1) is called a Type II limit solution if it is defined for $-\infty < t < \infty$ with uniformly bounded curvature, nonnegative holomorphic bisectional curvature and positive Ricci curvature where the scalar curvature R assumes its maximum in space-time.

Definition 1.2. A complete solution $g_{i\bar{j}}$ to Eq.(1.1) is called a Type III limit solution if it is defined for $0 < t < \infty$ with uniformly bounded curvature, nonnegative holomorphic bisectional curvature and positive Ricci curvature where tR assumes its maximum in space-time.

To understand the behavior of Type II or Type III limit solutions is very important because they arise as limits of blow ups of singularities in the Ricci flow, as pointed out by Richard Hamilton in [5].

In the Riemannian case, Hamilton [4] proved that any Type II limit with positive curvature operator is necessarily a gradient Ricci soliton.

Research partially supported by NSF grants # DMS-9307297 and # DMS-9504925.

Received December 12, 1995.