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HOMOLOGICAL REDUCTION OF CONSTRAINED POISSON ALGEBRAS

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Reduction of a Hamiltonian system with symmetry and/or constraints has a long history. There are several reduction procedures, all of which agree in "nice" cases [1]. Some have a geometric emphasis - reducing a (symplectic) space of states [39], while others are algebraic - reducing a (Poisson) algebra of observables [43]. Some start with a momentum map whose components are constraint functions [15]; some start with a gauge (symmetry) algebra whose generators, regarded as vector fields, correspond via the symplectic structure to constraints [10]. The relation between symmetry and constraints is particularly tight in the case Dirac calls "first class". The present paper is concerned entirely with this first class case and deals with the reduction of a Poisson algebra via homological methods, although there is considerable motivation from topology, particularly via the models central to rational homotopy theory.

Homological methods have become increasingly important in mathematical physics, especially field theory, over the last decade. In regard to constrained Hamiltonians, they came into focus with Henneaux's Report [22] on the work of Batalin, Fradkin and Vilkovisky [2], [3]-[5], emphasizing the acyclicity of a certain complex, later identified by Browning and McMullan as the Koszul complex of a regular ideal of constraints. I was able to put the BFV construction into the context of homological

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