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ON THE TOPOLOGICAL ENTROPY OF GEODESIC FLOWS

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1. Introduction

Let M^n be a closed connected C^∞ manifold and let SM be its unit tangent bundle, defined as usual as $SM = \{\theta = (x, v) : \ x \in M, \ v \in T_xM, \ ||v|| = 1\}$. The geodesic flow $\varphi_t : SM \to SM$ is defined by $\varphi_t(x, v) = (\gamma(t), \dot\gamma(t))$, where $\gamma : \mathbf{R} \to M$ is the geodesic with initial conditions $\gamma(0) = x$ and $\dot\gamma(0) = v$.

Given x and y in M, define $n_T(x, y)$ as the number of geodesics of length $\leq T$ (parametrized by arc length) joining x and y. A standard application of Sard's Theorem to the exponential maps of M shows that $n_T(x, y)$ is finite and locally constant on an open full measure subset of $M \times M$.

Our aim is to relate *the exponential growth rate of* $n_T(x, y)$, as a function of T, with the *topological entropy of the geodesic flow* $h_{top}(\varphi)$. In that direction, among other results, we shall prove that

$$h_{top}(\varphi) = \lim_{T \to +\infty} \frac{1}{T} \log \int_{M \times M} n_T(x, y)\, dxdy.$$

While proving this result, we shall also prove that *Przytycki's upper estimate* for the topological entropy of general C^2 flows [8], *is always an equality for* C^∞ *geodesic flows.* Since Przytycki's inequality will be a key tool in our proofs we begin by recalling its statement. Given a

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