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COUNTING PSEUDO-HOLOMORPHIC SUBMANIFOLDS IN DIMENSION 4

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The purpose of this article is to describe a certain invariant (called the Gromov invariant) for compact symplectic 4-manifolds which assigns an integer to each dimension 2-cohomology class. Roughly speaking, the invariant counts, with suitable weights, compact, pseudo-holomorphic submanifolds whose fundamental class is Poincaré dual to the cohomology class in question.

A version of this invariant was introduced originally by Gromov [2] to study the deformation classes of symplectic structures on manifolds with the homology of $\mathbb{CP}^2$. Subsequently, Ruan [7] extended Gromov's constructions to all symplectic 4-manifolds; the generalization of Ruan counts only connected, pseudo-holomorphic submanifolds. The invariant described below generalizes the construction of Ruan. The definition was sketched in [10] where the invariant was identified with the Seiberg-Witten invariants [13] of the symplectic manifold. However, the definition in [10] is incomplete in one respect - in its description of counting weights for multiply covered tori with trivial normal bundle. (The discussion in [7] is erroneous in this regard.) Thus, this article also serves to clear up any confusion stemming from counting these multiply covered tori.

Note that the equivalence claimed in [10] between the Seiberg- Witten invariant and the Gromov invariant as defined here holds for manifolds with $b^{2+} > 1$. The details of the proof will appear shortly (see [11], [12]).

This article is organized as follows: Section 1 defines the Gromov invariant as a weighted count of pseudo-holomorphic submanifolds. (See

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