J. DIFFERENTIAL GEOMETRY
Vol. 44 (1996) 789-806

ON THE ASYMPTOTIC CONE OF GROUPS SATISFYING A QUADRATIC ISOPERIMETRIC INEQUALITY

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Abstract

We prove that the asymptotic cone of a group satisfying a quadratic isoperimetric inequality is simply connected.

0. Introduction

The asymptotic cone of a group was introduced in [3], where it was used to prove that a group of polynomial growth is virtually nilpotent. It turns out that the group of isometries of the asymptoptic cone of a group of polynomial growth is a Lie group and plays a crucial role in Gromov's proof.

In [1] the construction of the asymptotic cone was generalized to arbitrary finitely generated groups. A complication appears in this case as one has to use ultrafilters in the definition, and it is not clear if the cone depends on the ultrafilter chosen. Because of this sometimes we will refer to all the asymptotic cones of a group as it is not known if this cone is unique. When on the other hand we speak of 'the' asymptotic cone of a group we mean that a specific ultrafilter has been fixed. It is known in many cases (e.g. for hyperbolic groups) that the cone is in fact independent of the ultrafilter .

In [5] Gromov relates the asymptotic cone of a group to the isoperimetric inequalities satisfied by the group. He proves that if every asymptotic cone of a group is simply connected, then the group satisfies a polynomial isoperimetric inequality. A more detailed exposition of this important result has been given by Drutu in [2]. Examples of groups

Received June 13, 1994, and, in revised form, March 6, 1995.