J. DIFFERENTIAL GEOMETRY
Vol. **44** (1996) 706-788

A PRODUCT FORMULA FOR THE SEIBERG-WITTEN INVARIANTS AND THE GENERALIZED THOM CONJECTURE

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1. Introduction

The Thom Conjecture asserts that any compact, embedded surface in $\mathbf{C}P^2$ of degree $d > 0$ must have genus at least as large as the smooth algebraic curve of the same degree, namely $(d-1)(d-2)/2$. More generally, one can ask whether in any algebraic surface a smooth algebraic curve is of minimal genus in its homology class. There was one significant result in this direction. Using $SU(2)$-Donaldson invariants, Kronheimer showed in [3] that this result is true for curves of positive self-intersection in a large class of simply connected surfaces with $b_2^+ > 1$. Unfortunately, for technical reasons, this argument does not extend to cover the case of $\mathbf{C}P^2$. It is the purpose of this paper to prove the general result that a smooth holomorphic curve of non-negative self-intersection in a compact Kähler manifold is genus minimizing.

For any closed, orientable riemann surface C we denote its genus by $g(C)$.

Theorem 1.1 (Generalized Thom Conjecture). *Let X be a compact Kähler surface and let $C \hookrightarrow X$ be a smooth holomorphic curve. Suppose that $C \cdot C \geq 0$. Let $C' \hookrightarrow X$ be a C^∞-embedding of a smooth riemann surface representing the same homology class as C. Then $g(C) \leq g(C')$.*

Received November 3, 1995. Partially Supported by NSF grants DMS 5244-58 and 9401714. The second author acknowledges the support of OTKA.