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SYMPLECTIC SUBMANIFOLDS AND ALMOST-COMPLEX GEOMETRY

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1. Introduction

In this paper we develop a general procedure for constructing symplectic submanifolds. Recall that if (V, ω) is a symplectic manifold, a submanifold $W \subset V$ is called symplectic if the restriction of ω to W is non-degenerate. Paradigms are complex submanifolds of complex Kähler manifolds. In general questions about complex submanifolds of high codimension can be intractable, but one has a rather good grip on submanifolds of complex codimension 1, which can be studied through the familar aparatus of line bundles, linear systems and cohomology—effectively linearising the problem. The idea of this paper is to extend these techniques in complex geometry to general symplectic manifolds. The main result we prove is the following existence theorem.

Theorem 1. *Let (V, ω) be a compact symplectic manifold of dimension $2n$, and suppose that the de Rham cohomology class $[\omega/2\pi] \in H^2(V; \mathbf{R})$ lies in the integral lattice $H^2(V; \mathbf{Z})/Torsion$. Let $h \in H^2(V; \mathbf{Z})$ be a lift of $[\omega/2\pi]$ to an integral class. Then for sufficiently large integers k the Poincaré dual of kh, in $H_{2n-2}(V; \mathbf{Z})$, can be realised by a symplectic submanifold $W \subset V$.*

In the case when V is a Kähler manifold and ω is the Kähler form, this reduces to a standard, but central, result in complex geometry. In that case one would argue that h is the first Chern class of a *positive line bundle* L over V, having a connection with curvature $-i\omega$, and then show that for large k the tensor power L^k has many holomorphic sections. The zero set of a generic section would provide the desired

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