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THE RADIUS RIGIDITY THEOREM FOR MANIFOLDS OF POSITIVE CURVATURE

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Abstract

Recall that the radius of a compact metric space $(X, dist)$ is given by $rad\ X = \min_{x\in X} \max_{y\in X} dist(x, y)$. In this paper we generalize Berger's $\frac{1}{4}$-pinched rigidity theorem and show that a closed, simply connected, Riemannian manifold with sectional curvature ≥ 1 and radius $\geq \frac{\pi}{2}$ is either homeomorphic to the sphere or isometric to a compact rank-one symmetric space.

The classical sphere theorem states that a complete, simply connected Riemannian n-manifold with positive, strictly 1/4-pinched sectional curvature is homeomorphic to S^n ([1], [16], and [21]). The weakly 1/4-pinched case is covered by

Berger's Rigidity Theorem ([2]). *Let M be a complete, simply connected Riemannian n-manifold with sectional curvature, $1 \leq sec\ M \leq 4$. Then either*

(i) *M is homeomorphic to S^n, or*

(ii) *M is isometric to a compact rank one symmetric space.*

The hypotheses of Berger's Theorem imply (with a lot of work) that the injectivity radius of M satisfies $inj\ M \geq \frac{\pi}{2}$ ([6] or [17]). The diameter therefore, also satisfies $diam\ M \geq \pi/2$, and the class of complete Riemannian manifolds with

$$(*) \qquad sec \geq 1 \text{ and } diam \geq \pi/2$$

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