A DECOMPOSITION OF SMOOTH SIMPLY-CONNECTED h-COBORDANT 4-MANIFOLDS

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Introduction and the Statement

In [1] S. Akbulut obtained an example of the exotic manifold by cutting off the contractible submanifold from the standard manifold and regluing it via nontrivial involution of the boundary.

In these notes we give a proof of a decomposition theorem stated below, which generalizes the example of Akbulut.

Another proof of the theorem was independently obtained by C.L. Curtis, M.H. Freedman, W.C. Hsiang, and R. Stong in [4].

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Throughout these notes all maps and manifolds are smooth, and immersions are in general position (or do their best if they have to obey some extra conditions). We also make the convention that if a star appears in place of a subindex, we consider a union of all objects in the family, where the index substituted by the star runs over its range. For example, $D_* \stackrel{\text{def}}{=} \bigcup_i D_i$.

Theorem. *Let U be a smooth, 5-dimensional, simply-connected h-cobordism with $\partial U = M_1 \sqcup (-M_2)$. Let $f : M_1 \to M_2$ be the homotopy equivalence induced by U. Then the following hold:*

1. There are decompositions

$$M_1 = M \natural_\Sigma W_1, \quad M_2 = M \natural_\Sigma W_2$$