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CURVY SLICING PROVES THAT TRIPLE JUNCTIONS LOCALLY MINIMIZE AREA

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Abstract

In soap films three minimal surfaces meet at 120-degree angles. We use a novel curvy slicing argument to prove that small pieces minimize area for given boundary. The argument applies in general dimension and codimension.

1. Introduction

The Belgian physicist J.A.F. Plateau [18] observed and recorded two types of singularities in soap films:

(1) three minimal surfaces meeting smoothly at angles of 120 degrees along a curve,

(2) four such curves meeting smoothly at angles of about 109 degrees at a point.

J. Taylor [20] proved that certain locally area-minimizing surfaces ($(\mathbf{M}, 0, \delta)$-minimal sets) have precisely these two types of singularities. For the first type, our Theorem 5.1 proves conversely that any such configuration is locally area minimizing in a stronger sense (among separators of regions). We extend our result to general dimension and codimension. Even for a regular piece of minimal surface, such area minimization is by no means obvious, and we begin with a new simple proof of this case (Theorem 2.1).

Whether "tetrahedral" singularities of type (2) locally minimize area remains an open question, although paired calibrations [LM] prove the tetrahedral cone itself minimizing in all dimensions. A more general conjecture states that a stationary singular surface minimizes area in

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