J. DIFFERENTIAL GEOMETRY
Vol. **44** (1996) 435-478

THE LARGE-SCALE GEOMETRY OF HILBERT MODULAR GROUPS

BENSON FARB & RICHARD SCHWARTZ

Abstract

Let G be the rank-2 semisimple Lie group $PSL_2(\boldsymbol{R}) \times PSL_2(\boldsymbol{R})$. In this paper we give a canonical isomorphism between the quasi-isometry group and the commensurator group of an irreducible, nonuniform lattice in G. The most familiar of these lattices are the classical Hilbert modular groups $PSL_2(\mathcal{O}_d)$, where $\mathcal{O}_d$ is the ring of integers in the real quadratic field $\boldsymbol{Q}(\sqrt{d})$. As corollaries to this theorem we obtain the following results:

1. The complete quasi-isometry classification of lattices in G.
2. Let Γ be any finitely generated group. If Γ is quasi-isometric to an irreducible, nonuniform lattice Λ in G, then Γ is a finite extension of an irreducible, nonuniform lattice commensurable with Λ in G.
3. Two irreducible, nonuniform lattices in G are quasi-isometric iff they are commensurable. In particular, no two distinct classical Hilbert modular groups are quasi-isometric.

1. Introduction

This paper introduces a new technique for studying rigidity of lattices in semisimple Lie groups, in particular for studying quasi-isometries without the usual equivariance assumptions as in Mostow Rigidity. The idea is to develope coarse metrical versions of some basic topological principles, for example Alexander duality and Jordan separation, and to apply these principles to pinning down the structure of quasi-isometries of a given lattice. This theory of "coarse topology" is developed and applied in §4 and §5. As a consequence we will prove the first known

Received September 20, 1995. Both authors are supported in part by N.S.F. Postdoctoral Fellowships.