J. DIFFERENTIAL GEOMETRY
44 (1996) 371-434

A BOUNDARY REGULARITY THEOREM FOR MEAN CURVATURE FLOW

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Abstract

We study singularity formation in the mean curvature flow of smooth, compact, embedded hypersurfaces of non-negative mean curvature in $\mathbf{R}^{n+1}$, with fixed smooth boundary, Γ. Then, subject to a so-called "Type I" hypothesis, and a certain geometrical constraint on Γ, we establish the following boundary regularity result:

Main Theorem (Boundary Regularity). *Suppose hypotheses A and B of Section 1 hold, and suppose that the hypersurfaces $\{M_t\}_{t\in[0,T)}$ are flowing by mean curvature as in (1.1). Then there is a fixed neighbourhood (in $\mathbf{R}^{n+1}$) of the boundary, Γ, in which all the surfaces M_t remain smooth, with uniform bounds on $|A|^2$ and all its derivatives, even as $t \nearrow T$.*

Note for instance that this result covers (modulo the Type I hypothesis) the model situation where Γ is some smooth (but possibly highly complicated) embedded submanifold of S^n which splits S^n into two "caps", and where M_0 is taken to be one of these caps (cf. Appendix C).

In the process of proving this theorem we also extend Huisken's classification of singularities (see [5]) to our setting with boundary, and refine his analysis along the lines of [10].

The principal ingredients used, to address these issues, are Allard's boundary regularity theory for varifolds, and also a certain "density function", whose definition is based on the analogue, for surfaces with boundary, of Huisken's important monotonicity formula for mean curvature flow.

Received May 2, 1994.