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A CONSTRUCTION OF SINGULAR SOLUTIONS FOR A SEMILINEAR ELLIPTIC EQUATION USING ASYMPTOTIC ANALYSIS

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Abstract

The aim of this paper is to prove the existence of weak solutions to the equation $\Delta u + u^p = 0$ which are positive in a domain $\Omega \subset \mathbb{R}^N$, vanish at the boundary, and have prescribed isolated singularities. The exponent p is required to lie in the interval $(N/(N-2), (N+2)/(N-2))$. We also prove the existence of solutions to the equation $\Delta u + u^p = 0$ which are positive in a domain $\Omega \subset \mathbb{R}^n$ and which are singular along arbitrary smooth k-dimensional submanifolds in the interior of these domains provided p lies in the interval $((n-k)/(n-k-2), (n-k+2)/(n-k-2))$. A particular case is when $p = (n+2)/(n-2)$, in which case solutions correspond to solutions of the singular Yamabe problem. The method used here is a mixture of different ingredients used by both authors in their separate constructions of solutions to the singular Yamabe problem, along with a new set of scaling techniques.

1. Introduction and statements of main results

In this paper we construct solutions with prescribed singularities for the semilinear elliptic equation $\Delta u + u^p = 0$ and other closely related equations, for a certain range of values of the exponent p, in a variety of situations. The solutions will have singularities prescribed along a disjoint union of submanifolds of varying dimension. We now describe our results, starting with the simplest case when the prescribed singular set is discrete.

Suppose Ω is any bounded open set in $\mathbb{R}^N$, with smooth boundary. Consider the equation

$$(1) \qquad \begin{cases} -\Delta u &= u^p \quad \text{in} \quad \Omega \\ \quad u &= 0 \quad \text{on} \quad \partial\Omega, \end{cases}$$

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