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GEOMETRIC EXPANSION OF CONVEX PLANE CURVES

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1. Introduction

In the last several years, there has been considerable interest in the deformation of Euclidean hypersurfaces in the direction of their normal vector field with speed various functions of the principal curvatures. In particular, for contracting flows, Gage-Hamilton [12] and Grayson [14] studied the curve shortening flow, Brakke [3] and Huisken [19] studied the mean curvature flow and Tso [22] studied the Gauss curvature flow (see also Chow [6], [7], Hamilton [17], [18] and Andrews [2]). For more general homogeneous contracting flows, see Andrews [1]. Besides contracting flows, there has also been recent interest in expanding flows. Similar results have been proved by Urbas [23], [24], Huisken [19] and Gerhardt [13]. More recently, Andrews [1] has studied more general expanding flows, especially flows with anisotropic speeds.

In each of the above papers, the hypersurfaces are evolving with speed a *homogeneous* increasing function of the principal radii. For expanding flows, one generally assumes in addition that the function is *concave.* In a series of papers, of which this is the first, we investigate expanding flows with speed an increasing function of the principal radii. In particular, we shall not assume the function is homogeneous. Our results generalize most of the previous results on expanding flows.

In this paper we study the motion of a smooth, strictly convex, embedded closed curve in $\mathbb{R}^2$ expanding in the direction of its outward normal vector with speed given by an *arbitrary* positive increasing function G of its principal radius of curvature. Our result is that there exists a unique one-parameter family of smooth, strictly convex curves

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