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A SPHERE THEOREM WITH A PINCHING CONSTANT BELOW $\frac{1}{4}$

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Dedicated to M. Berger

Abstract

In this article we establish a topological sphere theorem for compact, simply connected, odd dimensional manifolds M^n with pinched sectional curvature. The pinching constant δ_{odd} that appears in our hypotheses is $< \frac{1}{4}$, explicit, and independent of the dimension. Furthermore, in the even dimensional case we can find a pinching constant $\delta_{\text{ev}} < \frac{1}{4}$ which guarantees that the integral cohomology groups of M^n coincide up to torsion groups of odd order with the cohomology groups of a sphere or a projective space.
Both results are reduced by means of the diameter sphere theorem of Grove and Shiohama to proving Berger's horse shoe conjecture under a suitable condition on the diameter. The geometric arguments rely on refined Jacobi field estimates, which might be useful in other contexts as well.

Introduction

Berger's rigidity theorem provides a classification of all compact, simply connected manifolds M^n which carry a Riemannian metric with positive, weakly $\frac{1}{4}$–pinched sectional curvature [6]. This result has already been known in 1961. Then it has taken more than twenty years before the tools have been available to analyze weaker pinching conditions.

In fact, the pinching below–$\frac{1}{4}$ theorem for even dimensional manifolds [10] obtained by Berger in 1983 has been the first major application of

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