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ON A CONJECTURE OF CLEMENS ON RATIONAL CURVES ON HYPERSURFACES

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0. Introduction

In [2], H. Clemens proved the following theorem:

0.1 Theorem. *Let $X \subset \mathbb{P}^n$ be a general hypersurface of degree $d \geq 2n-1$. Then X contains no rational curve.*

In [3],[4] Ein generalized Clemens theorem in two directions; he considered a smooth projective variety M of dimension n, instead of $\mathbb{P}^n$ (which is a mild generalization since any such M can be projected to $\mathbb{P}^n$), and general complete intersections $X \subset M$ of type $(d_1, \dots, d_k)$ and proved:

0.2 Theorem. *If $d_1 + \dots + d_k \geq 2n - k - l + 1$, any subvariety Y of X of dimension l has a desingularisation $\tilde{Y}$ which has an effective canonical bundle; if the inequality is strict, the sections of $K_{\tilde{Y}}$ separate generic points of $\tilde{Y}$.*

In the case of divisors $Y \subset X$, this result has been improved by Xu [11],[12], who proved:

0.3 Theorem. *Let $Y \subset X$ be a divisor, $\tilde{Y}$ a desingularization of Y, then $p_g(\tilde{Y}) \geq n-1$ if $\sum d_i \geq n+2$.*

In [11], he gave more precise estimates for the minimal genus of a curve in a general surface in $\mathbb{P}^3$.

Now these results are not optimal, excepted in the case of divisors. In fact we will prove in the case of hypersurfaces the following improvement of Clemens and Ein's results:

0.4 Theorem. (See 2.10.) *Let $X \subset \mathbb{P}^n$ be a general hypersurface of degree $d \geq 2n - l - 1$, $1 \leq l \leq n-3$; then any subvariety Y of X of dimension l has a desingularization $\tilde{Y}$ with an effective canonical bundle; if the inequality is strict, the sections of $K_{\tilde{Y}}$ separate generic points of Y.*

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