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DIFFERENTIAL-GEOMETRIC CHARACTERIZATIONS OF COMPLETE INTERSECTIONS

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Abstract

We characterize complete interesections in terms of local differential geometry.

Let $X^n \subset \mathbb{CP}^{n+a}$ be a variety. We first localize the problem; we give a criterion for X to be a complete intersection that is testable at any smooth point of X. We rephrase the criterion in the language of projective differential geometry and derive a sufficient condition for X to be a complete intersection that is computable at a general point $x \in X$. The sufficient condition has a geometric interpretation in terms of restrictions on the spaces of osculating hypersurfaces at x. When this sufficient condition holds, we are able to define systems of partial differential equations that generalize the classical Monge equation that characterizes conic curves in $\mathbb{CP}^2$.

Using our sufficent condition, we show that if the ideal of X is generated by quadrics and $a < \frac{1}{3}[n-(b+1)+3]$, where $b = \dim X_{sing}$, then X is a complete intersection.

0. Introduction

Local and global geometry

Projective differential geometry has been used to study the local geometry of subvarieties of projective space by various authors (e.g. [2], [4], [6], [8], [15]). However, there are few examples where global conclusions are drawn from the local picture.

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